THE ROTATION OF THE ELECTROLYTE BETWEEN TWO COAXIAL CYLINDRICAL ELECTRODES IN THE MAGNETIC FIELD

Branislav Jovanovic, *Regional department in Kragujevac*

Teachers planning, teaching process, pupils' learning support, assessment as part of teaching, provision of resources are core issues of every teacher's activities that are non-static but evolving according the needs and directed towards the concept of permanent upgrading of teaching staff in the sense of knowledge, skills and attitudes, methodology, etc. The ground base for any activities is an in depth knowledge of the subject in this case the Physics. Teachers of Physics are expected to have also the skills of Physics phenomena model design and the ability to use computer simulation models.

Seen in the light of practical knowledge and skill transfer of specifically the Rotation of the electrolyte between two coaxial cylinders in the magnetic field, problem solving approach was analytical, computer simulation is a follow up that testifies the validity of the Mathematics model presented.

In the (cylindrical) enclosed tank, made of nonconducting material, there are two concentric metal cylinders with different radius. In an annulus between cylinders there is the fluid-10 pct solution $CuSO_4$ in water. When a voltage applied between electrodes, positive conduction ions will flow along radius, in the direction of the electrical field. Negative ions drift in the opposite direction. If the lines of applied external uniform magnetic field are parallel to the axis of the cylinders (perpendicular to velocity of an ion) then the magnetic force is acting on ions that are on radial electric currents, as shown in Figure 1. Fluids in the tank start to rotate with angular velocity ω.

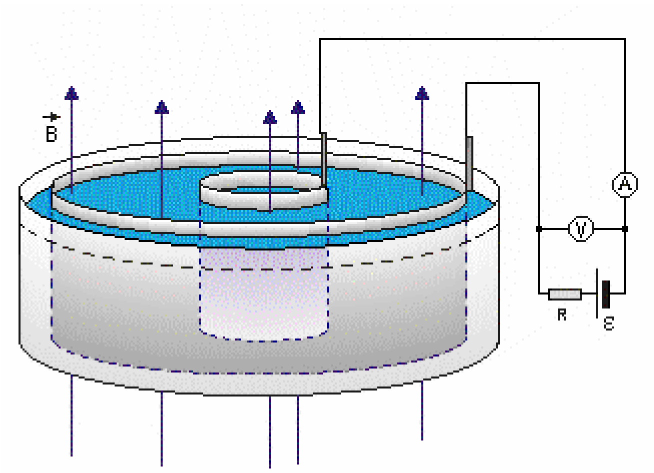

Figure 1 10% CuSO4 solution in water between two cylinders in a tank.

We suppose that the fluid is incompressible and Newtonian fluid. We also suppose that the velocity of the moving fluid at any fixed point does not change with time. If the magnitude of magnetic field B is constant and the path of the current I is not straight we can take a small straight segment along radius (the layer) width dr and write for tangentional component of magnetic force:

$$dF_T = I \cdot dr \cdot B \quad (1)$$

Let us take Cartesian coordinate system where z-axis is identical with cylindrical axis. If contact surface of layers is A, dynamic viscosity η, x and y components of layer velocity v_x and v_y, frictional force is:

$$dF = \eta \cdot A \cdot \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) \quad (2)$$

Using the relations for Cartesian coordinates: x, y and polar coordinates: radius r, angle $\theta = \omega t$ then $x = r \cos \omega t$, $y = r \sin \omega t$ and for $v_x = - \omega y$, $v_y = \omega x$, we can obtain equation (2) in polar coordinate system:

$$dF = \eta \cdot r \cdot A \cdot \frac{d\omega}{dr} \quad (3)$$

Viscous torque, the resistance that a fluid offers to rotational motion is proportional to angular velocity gradients:

$$M = \eta \cdot r^2 \cdot A \cdot \frac{d\omega}{dr} \quad (4)$$

Equation (4) which gives the viscous torque of a fluid in pure rotation is the angular equivalent of the Newton's law, which gives the viscous force of a fluid in pure translation. This is the rotational analog of Newton's law of the viscous force, where dynamic viscosity η, appears in Newton's law, ηr^2 appears in formula (4).

The magnetic force (1) and difference viscous forces between two layers surfaces of fluid, radii r and $r+dr$, (3), plotted in Figure 2, are in balance:

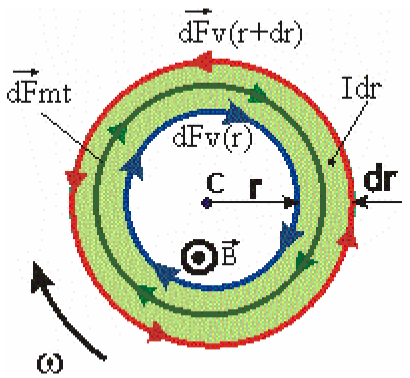

Figure 2 The magnetic force and difference viscous forces between two layers surfaces of fluid.

$$d\left(\eta \cdot r \cdot A \cdot \frac{d\omega}{dr} \right) = I \cdot B \cdot dr \quad (5)$$

After substitute in (5) surface $A = 2\pi r h$, where h is fluid depth, and taking the derivative, this equation becomes differential equation:

$$2\pi \cdot r \cdot h \cdot \left(2 \cdot r \cdot \frac{d\omega}{dr} + r^2 \cdot \frac{d^2\omega}{dr^2} \right) \cdot dr = I \cdot B \cdot dr \quad (6)$$

For $r > 0$, (evidently) we take e^s instead r, and that gives:

$$\frac{d^2\omega}{ds^2} + \frac{d\omega}{ds} = \frac{I \cdot B}{2\pi \cdot \eta \cdot h} \tag{7}$$

Plugging:

$$\omega = k + \frac{I \cdot B}{2\pi \cdot \eta \cdot h} \cdot s + C \tag{8}$$

where C is the constant, into (7) gives simply homogeneous differential equation. On condition that $\omega = 0$ for $r = r_o$ and $r = R$, where r_o and R are cylinders radius, we obtain relation for angular velocity:

$$\omega = \frac{I \cdot B}{2\pi \cdot \eta \cdot h} \cdot \left[\ln\frac{R}{r} - \frac{r_o}{R - r_o} \cdot \left(\frac{R}{r} - 1 \right) \cdot \ln\frac{R}{r_o} \right] \tag{9}$$

It is interesting to realize that the motion of an ion can be considered as superposition of the term $v_r\ t$, which arises from electric force and radial component magnetic force, and the term $v_t\ t$, wich arises from the tangential component magnetic force and rotation of the fluid. V_r and v_t are radial and tangential components of the average ion velocity:

$$v_r = \mu \cdot (E - v_t \cdot B) \tag{10}$$

$$v_t = \mu \cdot E_t + \omega \cdot r = \mu \cdot v_r \cdot B + \omega \cdot r \tag{11}$$

In the above equation the following stands for: E - radial electric field, μ - mobility of ion and $E_t = v_r B$ - equivalent tangential electric field. From this expression, we can find as follows:

$$v_r = \frac{\mu \cdot (E - \omega \cdot r \cdot B)}{1 + \mu^2 \cdot B^2} \tag{12}$$

We now can return this relation in (11) and obtain again v_t but μ is so small, $\mu^2 B^2 << 1$.

Certainly, we now relate potential difference between the cylinders, U, to the radial electric field E by:

$$E = \frac{U}{\ln\frac{R}{r_o}} \cdot \frac{1}{r} \tag{13}$$

A two-dimensional model was completed and used to illustrate the motion of ions between two cylinders. When such situations arise, we can use Euler method for numerical modeling to study motion of ions in fluid. If the angular velocity at any instant t is known, the ion's velocity and position, in the polar coordinates, at time $t + \Delta t$ (for a small increment of time Δt) can be calculated from equations:

1. $$\omega = \frac{I \cdot B}{2\pi \cdot \eta \cdot h} \cdot \left[\ln \frac{R}{r} - \frac{r_o}{R - r_o} \cdot \left(\frac{R}{r} - 1 \right) \cdot \ln \frac{R}{r_o} \right]$$

2. $$v_r = \frac{\mu \cdot (E - \omega \cdot r \cdot B)}{1 + \mu^2 \cdot B^2}$$

3. $$v_t = \mu \cdot v_r \cdot B + \omega \cdot r$$

4. $\Delta\tau = v_t \cdot \Delta t$, tangentional displacement,

5. $\theta = \theta - \frac{\Delta\tau}{r}$,the angle between r and fixed axis x,

6. $r(t + \Delta t) = r(t) + v_r \cdot \Delta t$

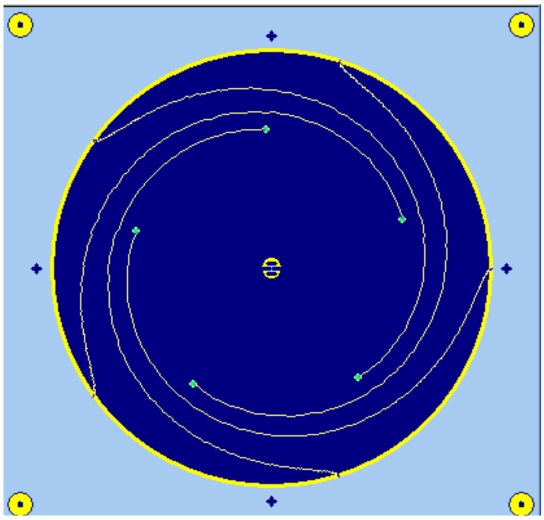

Figure 3 Position-time graph for the motion of the small circles as positive ions in a fluid, from outer cylinder to inner.

The calculation then proceeds in a series of finite steps to determine the angular velocity, components of velocity, angle and radius at any later time. The calculations carried out by using a program written in DELPHI (Kylix), Figures 3. Program allows calculating, among others, ion parameters as distance traveled, the total time the ion spends in the electrolyte and the rotation angle path of ions depending from r_o, R, and B. We also can display graph of linear velocity components and angular velocity, shown in Figure 4. If *U* is constant when radii r_o and *R* change then we can anticipate approximately that current is also constant.

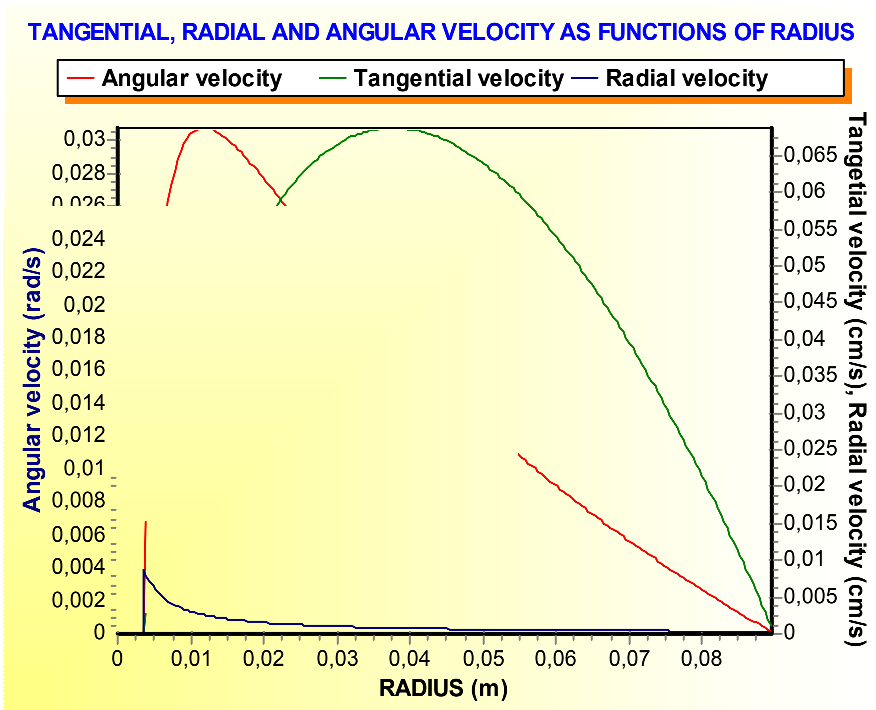

Figure 4 B=10^{-5} T, ro=3,5mm, R=9cm, μ= 10^{-7} m^2/(V s), η=10^{-3} kg/(m s), h=0,02m, I=1,33A, U=10V. Tangential, radial and angular velocity as functions of radius.

e.g. When the magnitude of magnetic field is B=10^{-5}T, ro=3,5mm, R=9cm, μ=$10^{-7}$$m^2$/(V s), η=$10^{-3}$ kg/(m s), h=0,02m, I=0,33A, U=10V then program gives: the distance traveled = 5,6m, the total time=13130s, the ions sweeps out an angle=7444°, the average of radial velocity = 6,6 μm/s and the average of tangential velocity = 0,43 mm/s .

The described model shows the movement of ions in the presence of both an electric field and magnetic field. We can demonstrate magnetic force, electrical force and viscouse force. Simulation of this dynamic process using computers in the teaching-learning process is needed to visualize and calculate characteristics of the phenomena. Modeling of the process enables us to formulate the problem, mathematical model design, model testing, researches, comparison of the experimental results with computers results, (further model development), deepening the views of Physics laws and testing the limits related to the law application.

Using computer simulation of Physics process we would also like to develop an example of application that will help teachers and pupils to understand Physics laws and phenomena, better.

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